Direction of Arrival Estimation (DOA) in Interference & Multipath Propagation

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Scope

- There are many location methods
  - Source location vs. Self-location (Navigation)
  - Active vs. Passive
  - Network based (GPS) vs. Single platform (DOA or AOA)
- We will concentrate on source location with a single platform equipped with a sensor array, passive
- Emphasis on DOA estimation
- Narrowband signals only

Outline

- Basics of DOA Estimation
  - Beamforming Type
  - High Resolution Type
- Adaptive Beamforming DOA Estimation in Strong Interference
- High Resolution DOA Estimation in Multipath
  - Smoothing Method
- Source Association and Locating the Sources

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- Basics of DOA Estimation
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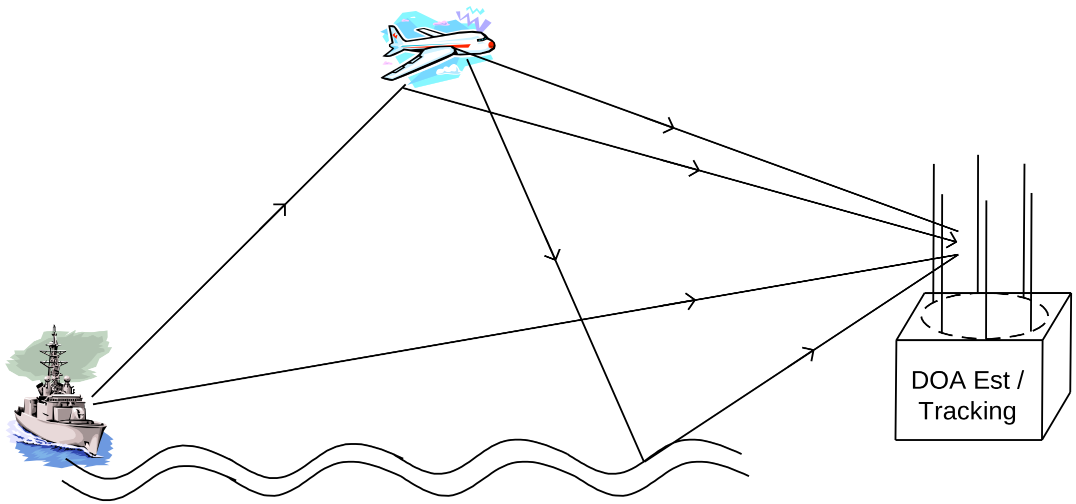
Objective of DOA Estimation

- To find DOA (relative to the array orientation) of all incident RF rays
- Could have multipath propagation and interference



Narrowband Signal Sources

- A complex sinusoid

$$s(t) = \alpha e^{j\beta} e^{j\omega t} = \rho e^{j\omega t}$$

- A real sinusoid is a sum of two sinusoids

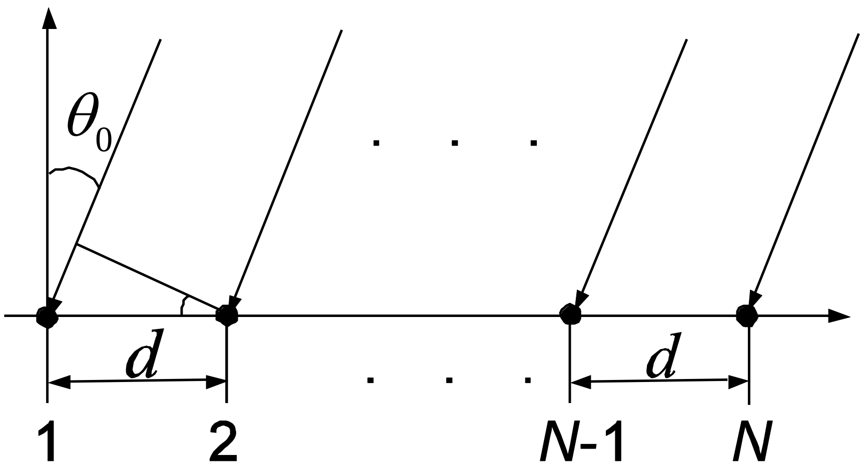
$$\alpha \cos(\omega t + \beta) = \frac{\alpha}{2} e^{j\beta} e^{j\omega t} + \frac{\alpha}{2} e^{-j\beta} e^{-j\omega t} = \rho_1 e^{j\omega t} + \rho_2 e^{-j\omega t}$$

- A delay of a sinusoid is a phase shift

$$s(t - t_0) = e^{-j\omega t_0} \rho e^{j\omega t} = e^{-j\omega t_0} s(t)$$

- Apply approximately to narrowband signals

A Uniform Linear Array



A signal source $s(t) = \rho e^{j\omega t}$

"impinges" on the array

with an angle θ_0

c: propagation speed

- If the received signal at sensor 1 is $x_1(t) = s(t)$
- Then it is delayed at sensor i by $\Delta_i = \dfrac{(i-1)d\sin\theta_0}{c}$
- Then the received signal at sensor i is

$$x_i(t) = e^{-j\omega\Delta_i} x_1(t) = e^{-j\omega\Delta_i} s(t) = e^{-j\omega\frac{(i-1)d\sin\theta_0}{c}} s(t)$$

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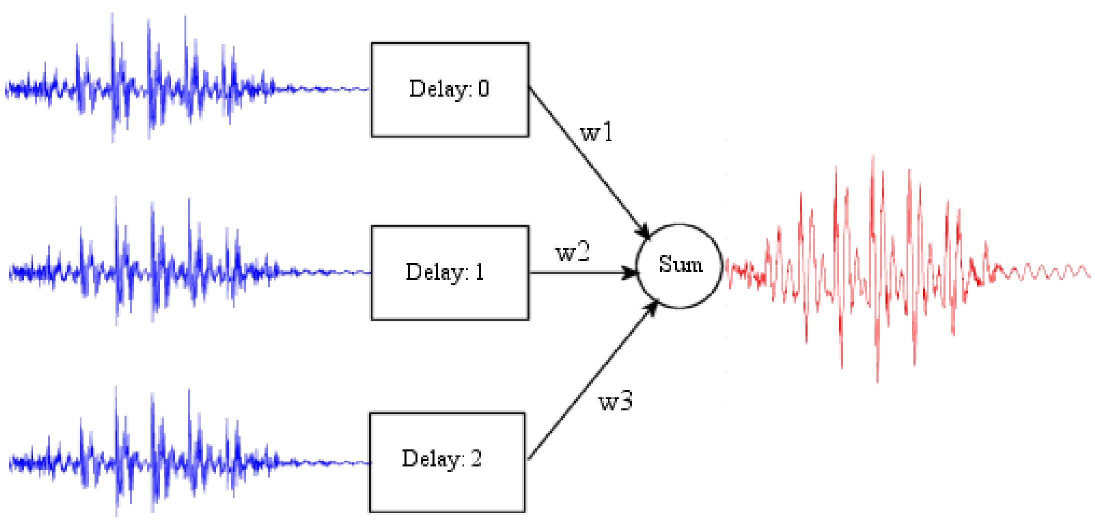
Beamforming

- Delay-and-sum type
- Reverse the delay/phase on each sensor to “line-up” the received signal phase
- Adding all phase-shifted sensor outputs to enhance the received SNR in that direction



Beamforming

- By adjusting the delay or phase shifts, we electronically steer the beam through all look directions to find DOA of an incident signal
- Received signal at the *i*th sensor:

$$x_i(t) = e^{-j\omega\Delta_i} x_1(t) = e^{-j\omega\Delta_i} s(t) = e^{-j\omega\frac{(i-1)d\sin\theta_0}{c}} s(t)$$

- θ_0 is the incidence angle of the received signal
- The delay or phase shift of the beamformer on the *i*th sensor is computed according to a "look angle" θ to get the output:

$$y(t) = \sum_{i=1}^{N} w_i^* x_i(t) = \sum_{i=1}^{N} e^{j\omega\frac{(i-1)d\sin\theta}{c}} x_i(t)$$

$$= \sum_{i=1}^{N} e^{j\omega\frac{(i-1)d[\sin\theta-\sin\theta_0]}{c}} s(t)$$

Beamforming

- If $\theta = \theta_0$, then the output becomes

$$y(t)\big|_{\theta=\theta_0} = \sum_{i=1}^{N} e^{j\omega\frac{(i-1)d[\sin\theta - \sin\theta_0]}{c}} s(t)\Bigg|_{\theta=\theta_0} = \sum_{i=1}^{N} s(t) = Ns(t)$$

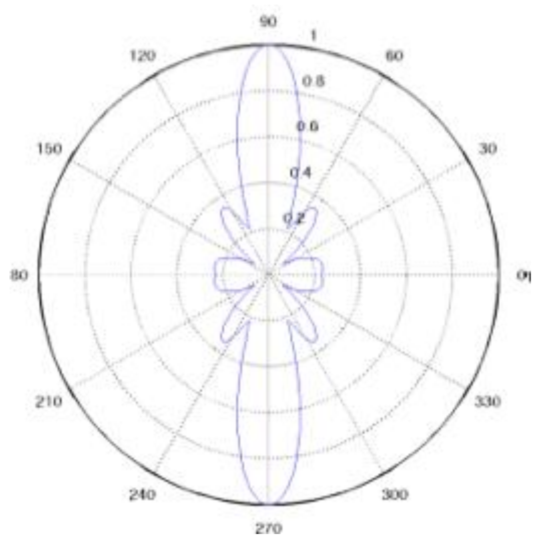
- I.e., the output is enhanced N times
- Since noise is not correlated, noise is not enhanced
- So SNR is enhanced N times
- At other steering angles, the complex weights may cancel so the output is not enhanced or even degraded
- This is the designed purpose of a beamformer

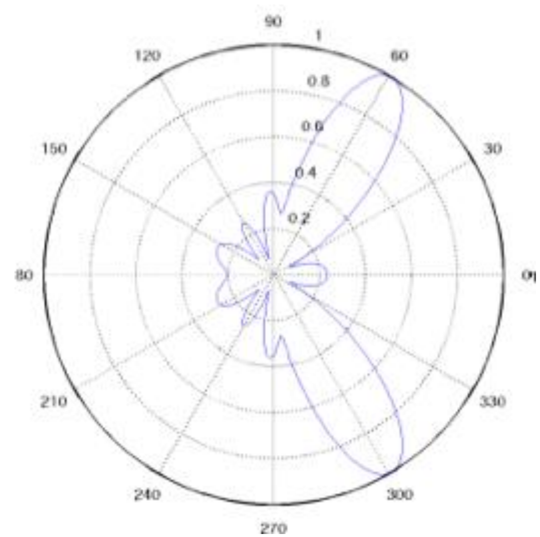
Beamforming

- Beam pattern: A gain pattern as a function of the steering angle

$$G(\theta,\theta_0) = \left| \sum_{i=1}^{N} e^{j\omega \frac{(i-1)d[\sin\theta - \sin\theta_0]}{c}} \right|$$

$\theta_0 = 0$

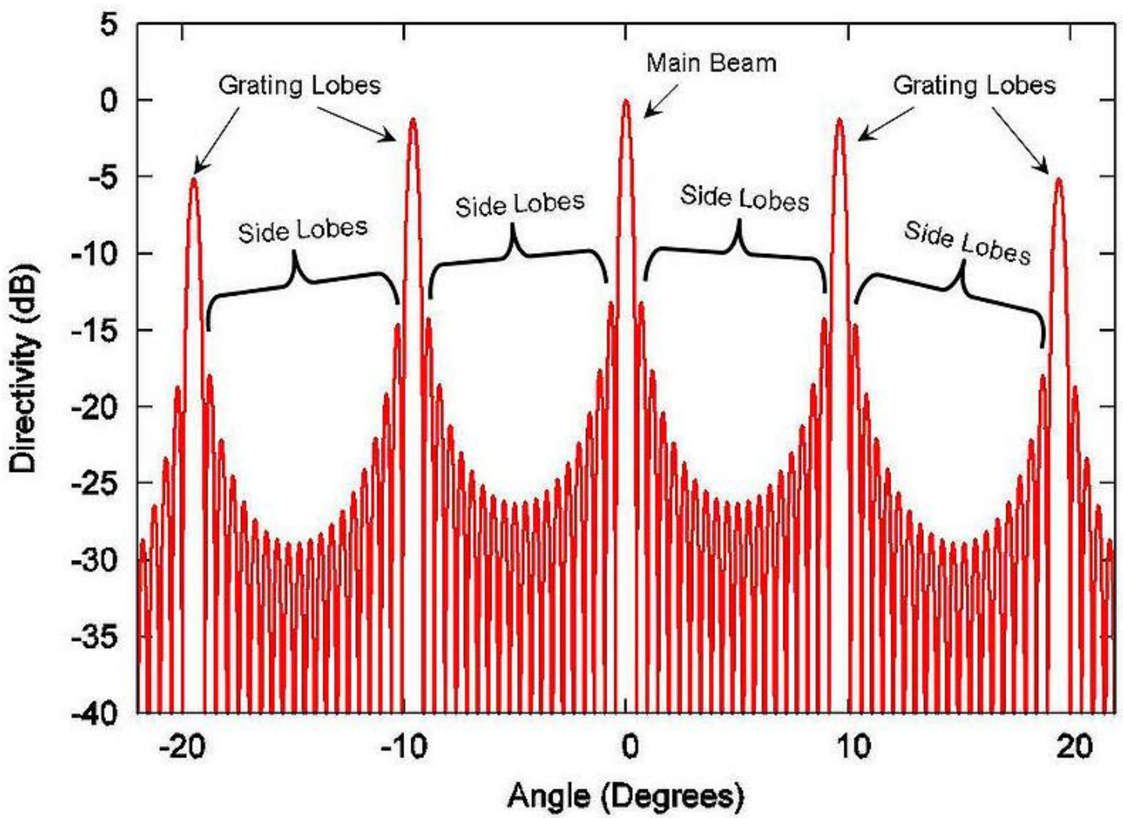




$\theta_0 = 30^{\circ}$

Beamforming

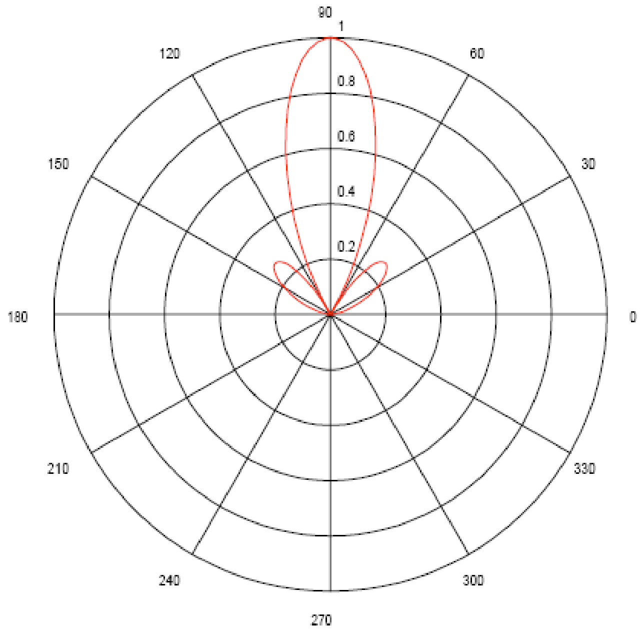
- Sensor spacing is usually wavelength/2
- Larger spacing creates "grating lobes" that confuse with the main lobe
- Smaller spacing reduces the total aperture – lower spatial resolution



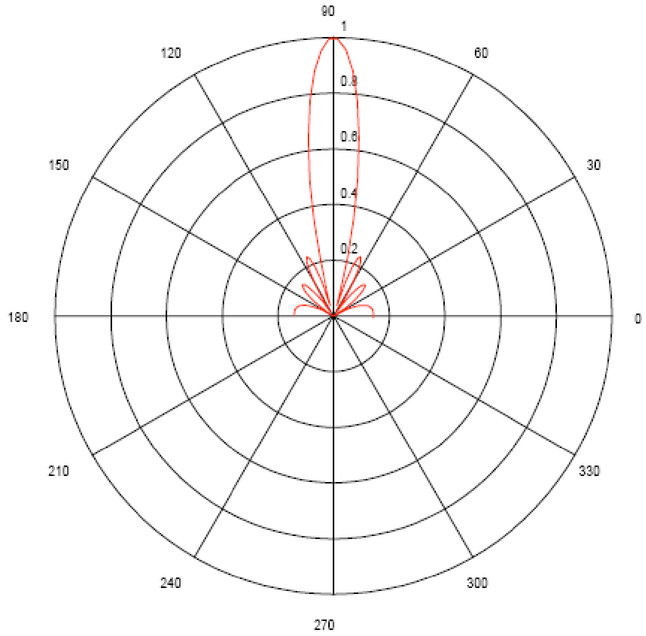
Beamforming

- Number of sensors (array aperture) are directly related to spatial resolution

4-element (aperture = 1.5 wavelength)



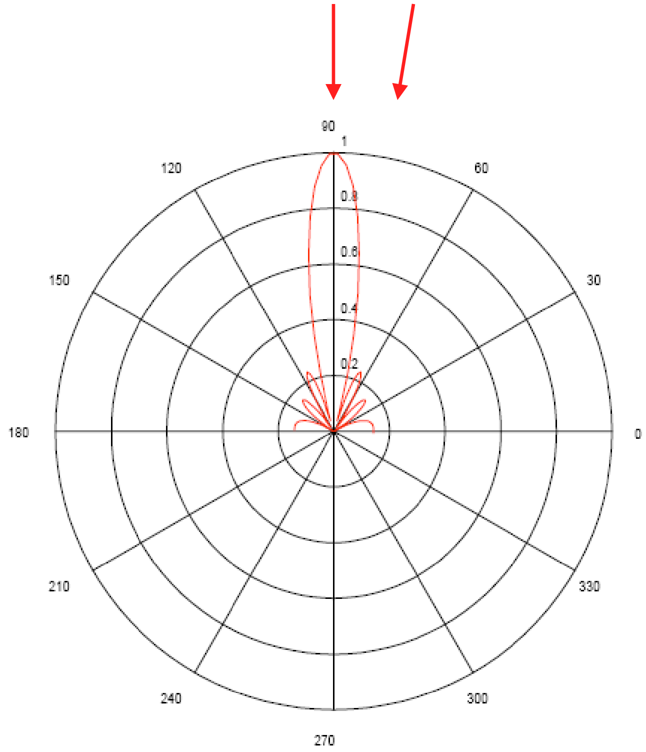
7-element (aperture = 3 wavelength)



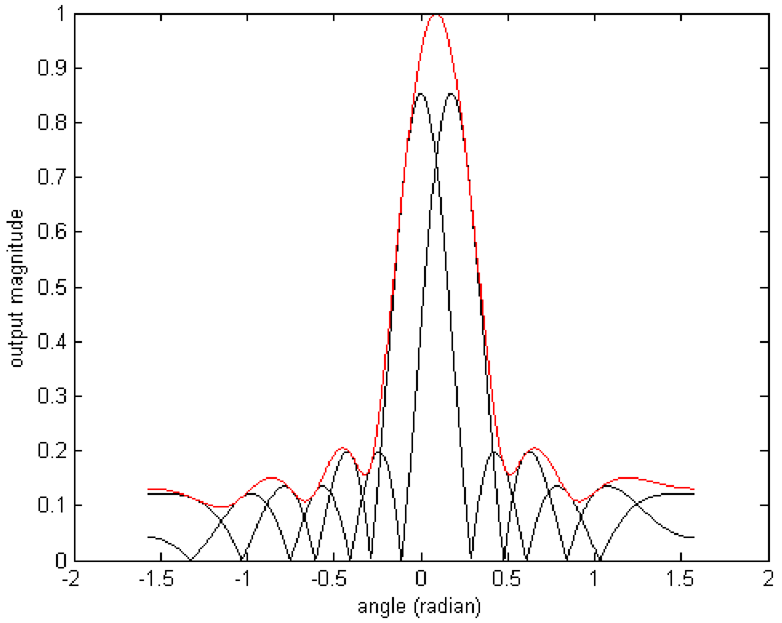
Beamforming

- Problem 1: A wide mainlobe causes poor spatial resolution

Two targets 10 degrees apart

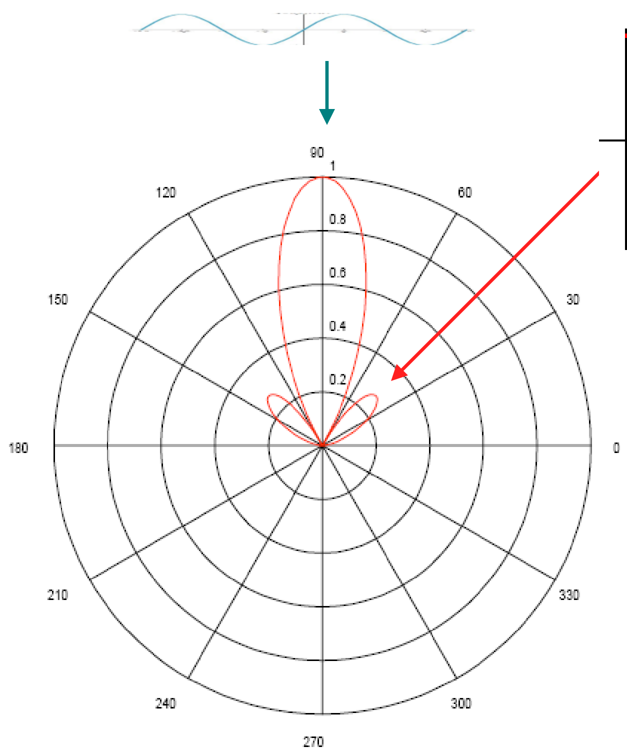


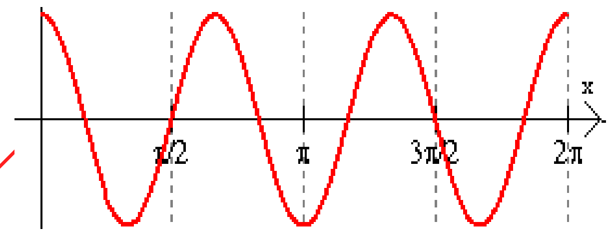
Cannot distinguish them



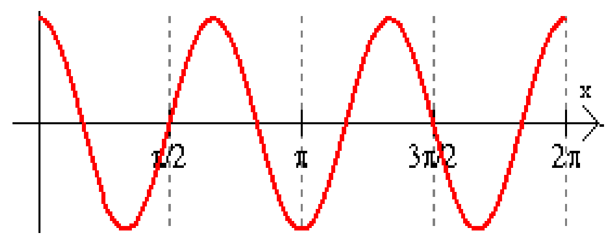
Beamforming

- Problem 2: A strong interferer can come into a sidelobe and completely swamp weak signal in the look direction





Beamformer output:



- Control sidelobes by Dolph-Chebyshev shading, w. limited effect

Pros/Cons of Beamforming

- Can find only one DOA at a time
- Spatial resolution determined by number of sensors in an array, but generally not very good unless having a large number of sensors
- Works for other array shapes also, need to know sensor positions in an array
- Sensitive to sensor position, gain, and phase errors, must calibrate carefully to make it work well
- Interference is a big problem
- Multipath is a much lesser problem

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MUSIC

- Able to find DOAs of multiple sources in “one-shot”
- High spatial resolution compared with beamforming methods (I.e., a few antennas can result in very high spatial resolution)
- MUSIC stands for Multiple Signal Classifier

Narrowband Signal Sources

- Consider *I* narrowband signal sources

$$s_1(t) = \rho_1 e^{j\omega_1 t}, \quad s_2(t) = \rho_2 e^{j\omega_2 t}, \quad \cdots, \quad s_I(t) = \rho_I e^{j\omega_I t}$$

- Assume that all amplitudes are uncorrelated

$$E\{\rho_i \rho_j\} = \begin{cases} \sigma_i^2; & i = j \\ 0; & i \neq j \end{cases}$$

- Recall received signal on the *i*th sensor:

$$x_i(t) = e^{-j\omega\Delta_i} x_1(t) = e^{-j\omega\Delta_i} s(t) = e^{-j\omega\frac{(i-1)d\sin\theta_0}{c}} s(t)$$

Signal Model

- Put received signals at all N sensors together:

$$\mathbf{x}(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_N(t) \end{bmatrix}=\begin{bmatrix} 1 \\ e^{-j\omega\frac{d\sin\theta}{c}} \\ e^{-j\omega\frac{2d\sin\theta}{c}} \\ \vdots \\ e^{-j\omega\frac{(N-1)d\sin\theta}{c}} \end{bmatrix} s(t)=\mathbf{a}(\theta)s(t)$$

- $\mathbf{a}(\theta)$ is called a "steering vector"

Signal Model

- If there are I source signals received by the array, we get a "signal model":

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$

$\mathbf{x}(t)$ --- received signal vector (N by 1)
$\mathbf{s}(t)$ --- source signal vector (I by 1)
$\mathbf{n}(t)$ --- noise vector (N by 1)
$\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_I)]$ (N by I)
$\mathbf{s}(t) = [s_1(t), \cdots, s_I(t)]^T$

- Sources are independent, noises are uncorrelated
- Column of **A** can also be normalized

The MUSIC Algorithm

- Compute the N x N correlation matrix

$$\mathbf{R_x} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}\mathbf{R_s}\mathbf{A}^H + \sigma_0^2\mathbf{I}$$

where $\mathbf{R_s} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\} = diag.\{\sigma_1^2, \cdots, \sigma_I^2\}$

- If the sources are somewhat correlated so $\mathbf{R_s}$ is not diagonal, it will still work if $\mathbf{R_s}$ has full rank.
- If the sources are correlated such that $\mathbf{R_s}$ is rank deficient, then it is a problem. A common solution is "spatial smoothing".

Q: Why is the rank of $\mathbf{R_s}$ (being I) so important?

A: It defines the dimension of the signal subspace.

The MUSIC Algorithm

- For $N > I$, the matrix $\mathbf{AR_sA}^H$ is singular, i.e.,

$$\det[\mathbf{AR_sA}^H] = \det[\mathbf{R_x} - \sigma_0^2\mathbf{I}] = 0$$

- But this implies that σ_0^2 is an eigenvalue of $\mathbf{R_x}$
- Since the dimension of the null space of $\mathbf{AR_sA}^H$ is N-I, there are N-I such eigenvalues σ_0^2 of $\mathbf{R_x}$
- Since $\mathbf{R_x}$ is non-negative definite, there are I other eigenvalues σ_i^2 such that $\sigma_i^2 > \sigma_0^2 > 0$
- Let $\mathbf{u}_i$ be the ith eigenvector of $\mathbf{R_x}$ corresponding to σ_i^2

$$\mathbf{R_x u}_i = \sigma_i^2 \mathbf{u}_i; \quad i = 1, 2, \cdots, N$$

$$\sigma_i^2 > \sigma_0^2,\ i = 1, \cdots, I; \quad \sigma_i^2 = \sigma_0^2,\ i = I+1, \cdots, N$$

The MUSIC Algorithm

$$\mathbf{R_x u}_i = [\mathbf{A R_s A}^H + \sigma_0^2 \mathbf{I}]\mathbf{u}_i = \sigma_i^2 \mathbf{u}_i; \quad i = 1, 2, \cdots, N$$

- This implies

$$\mathbf{A R_s A}^H \mathbf{u}_i = (\sigma_i^2 - \sigma_0^2)\mathbf{u}_i; \quad i = 1, 2, \cdots, N$$

$$\mathbf{A R_s A}^H \mathbf{u}_i = \begin{cases} (\sigma_i^2 - \sigma_0^2)\mathbf{u}_i; & i = 1, 2, \cdots, I \\ 0; & i = I+1, \cdots, N \end{cases}$$

- Partition the *N*-dimensional vector space into the signal subspace $\mathbf{U_s}$ and the noise subspace $\mathbf{U_n}$

$$\begin{bmatrix} \mathbf{U_s} & \mathbf{U_n} \end{bmatrix} = [\underbrace{\mathbf{u}_1 \quad \cdots \quad \mathbf{u}_I}_{\mathbf{U_s}:\, \sigma_i^2,\ i \le I} \quad \underbrace{\mathbf{u}_{I+1} \quad \cdots \quad \mathbf{u}_N}_{\mathbf{U_n}:\, \sigma_i^2 = \sigma_0^2,\ i > I}]$$

The MUSIC Algorithm

- The steering vector $\mathbf{a}(\theta_i)$ is in the signal subspace
- Signal subspace is orthogonal to noise subspace

$$\mathbf{A}\mathbf{R}_{\mathbf{s}}\mathbf{A}^{H}\mathbf{u}_i = \begin{cases} (\sigma_i^2 - \sigma_0^2)\mathbf{u}_i; & i = 1, 2, \cdots, I \quad (1) \\ 0; & i = I+1, \cdots, N \quad (2) \end{cases}$$

(1) means I linear combinations of columns of **A** equal the signal subspace spanned by columns of $\mathbf{U}_{\mathbf{s}}$

(2) means the linear combinations of columns of **A**, i.e., the signal subspace, is orthogonal to $\mathbf{U}_{\mathbf{n}}$

The MUSIC Algorithm

- The steering vector $\mathbf{a}(\theta_i)$ is in the signal subspace
- Signal subspace is orthogonal to noise subspace
- This implies that $\mathbf{a}^H(\theta_i)\mathbf{U_n} = \mathbf{0}$
- So the MUSIC algorithm searches through all angles θ, and plots the "spatial spectrum"

$$P(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{U_n}}$$

- Wherever $\theta = \theta_i$, $P(\theta)$ exhibits a peak
- Peak detection will give spatial angles of all incident sources

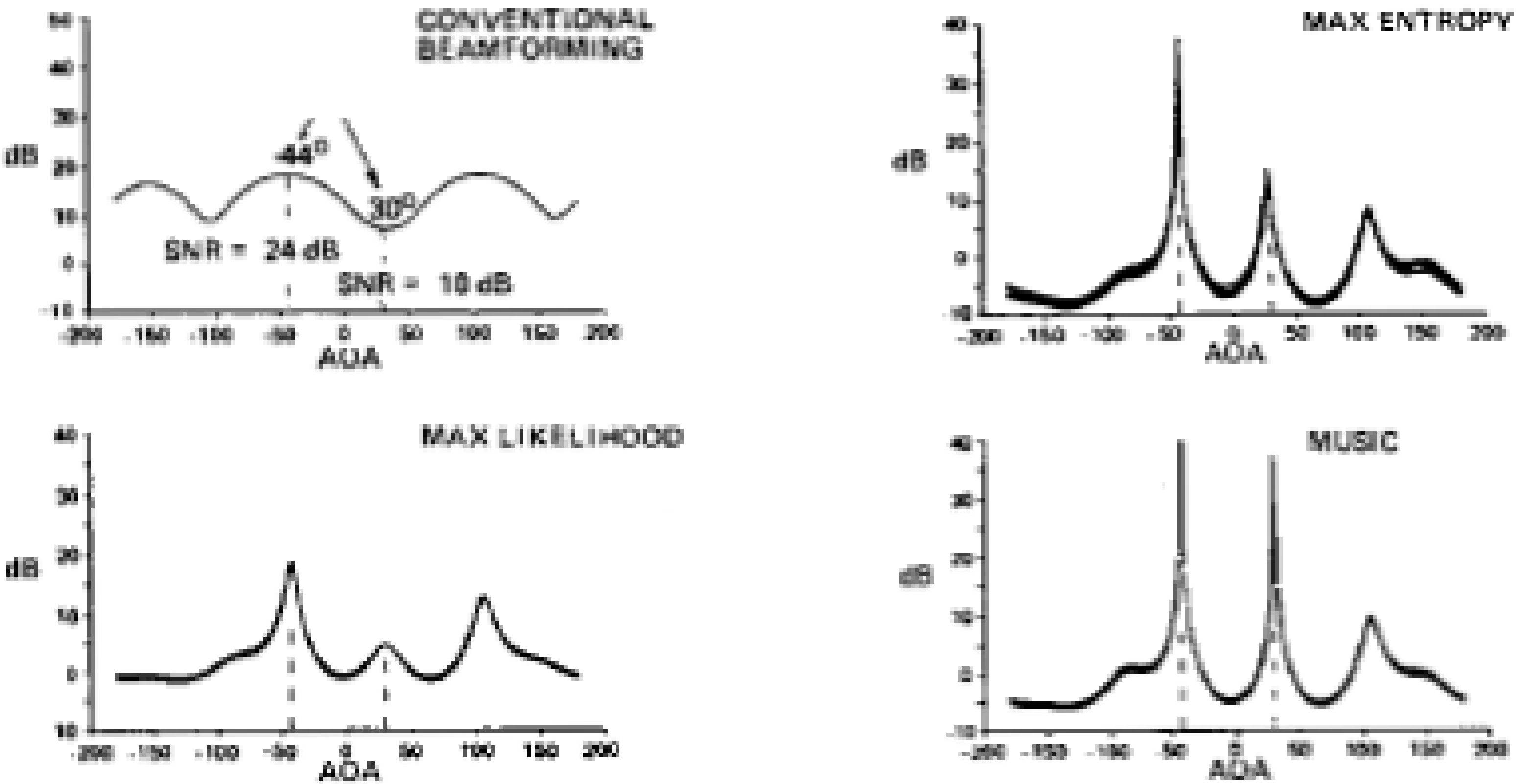
The MUSIC Algorithm

1. Compute the signal correlation matrix $\mathbf{R_x}$
2. Perform SVD/EVD on $\mathbf{R_x}$ and separate the smallest eigenvalues from larger eigenvalues
3. Eigenvectors corresponding to the smallest eigenvalues form a noise subspace $\mathbf{U_n}$
4. Search through all angles θ in the MUSIC spatial spectrum

$$P(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{U_n}}$$

5. Peaks correspond to DOAs

The MUSIC Algorithm



MUSIC spatial spectrum compared with other methods

Pros/Cons of The MUSIC Algorithm

- Can find multiple DOAs with high resolution
- Number of sensors must be more than number of sources
- Works for other array shapes also, need to know sensor positions in an array
- Very sensitive to sensor position, gain, and phase errors, need careful calibration to make it work well
- Searching through all θ could be computationally expensive
- Interference is not much a problem, just another source whose DOA can be found with other sources
- Multipath is a big problem

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Adaptive Beamforming

- Rather than fixed weights/phase, adapt the weights according to the signal environment
- The generalized sidelobe canceller (GSC) achieves this w. flexible constraints
- Delay-and-sum beamformer output $y(t)$:

$$y(t) = \mathbf{a}^H(\theta)\mathbf{x}(t) = \mathbf{a}^H(\theta)\mathbf{a}(\theta_0)s(t)$$

- The GSC output $y(t)$:

$$y(t) = \mathbf{w}^H\mathbf{x}(t) = \mathbf{w}^H\mathbf{a}(\theta_0)s(t)$$

w is a vector of complex weights, more general

Adaptive Beamforming

- Q: How do we choose the weights **w**?
- A: By some constrained optimization
- Constraints:

1) Array gain at the look direction should be preserved

$$y(t) = \mathbf{w}^H \mathbf{x}(t) = \mathbf{w}^H \mathbf{a}(\theta_0) s(t)$$

$$\mathbf{w}^H \mathbf{a}(\theta_1) = \mathbf{a}^H(\theta_1)\mathbf{w} = 1$$

When $\theta_1 = \theta_0$, we have $y(t) = s(t)$

2) Array gain at some other directions may need to be zero

$$\mathbf{w}^H \mathbf{a}(\theta_2) = \mathbf{a}^H(\theta_2)\mathbf{w} = 0$$

This is also called null-steering

Adaptive Beamforming

- Jointly, we write these constraints as

$$\begin{bmatrix} \mathbf{a}^H(\theta_1) \\ \mathbf{a}^H(\theta_2) \end{bmatrix} \mathbf{w} = \left[\mathbf{a}(\theta_1) \quad \mathbf{a}(\theta_2)\right]^H \mathbf{w} = \mathbf{C}^H \mathbf{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \mathbf{g}$$

- More constraints, other constraints, can be imposed
- The cost function is:

$$E\{|y(t)|^2\} = \mathbf{w}^H E\{\mathbf{x}(t)\mathbf{x}^H(t)\}\mathbf{w} = \mathbf{w}^H \mathbf{R}_\mathbf{x} \mathbf{w}$$

- The optimization problem is:

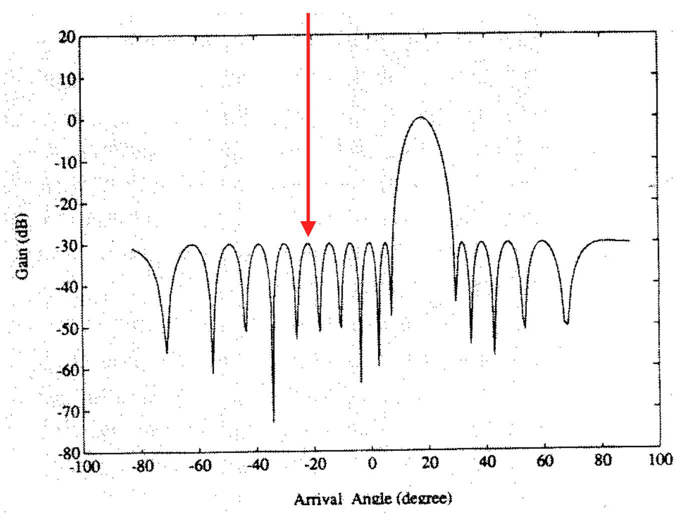
$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_\mathbf{x} \mathbf{w} \quad \text{subject to} \quad \mathbf{C}^H \mathbf{w} = \mathbf{g}$$

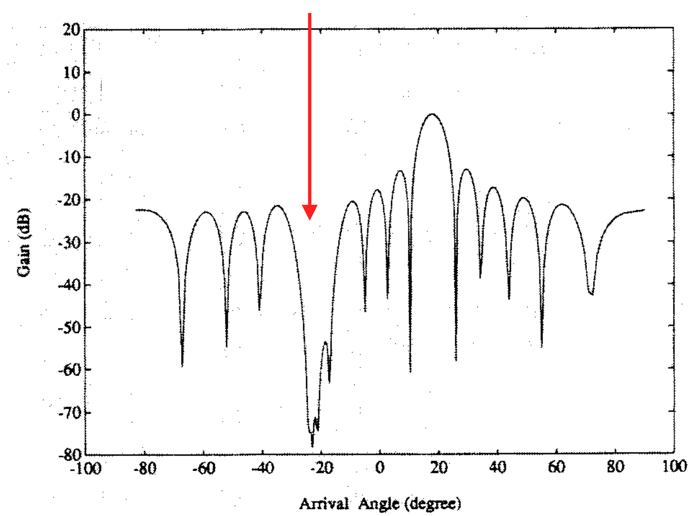
Adaptive Beamforming

- The solution to this optimization problem is:

$$\mathbf{w} = \mathbf{R}_{\mathbf{x}}^{-1}\mathbf{C}(\mathbf{C}^{H}\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{C})^{-1}\mathbf{g}$$

- Since we minimize the output energy, a strong interferer will result in a deep notch in its direction



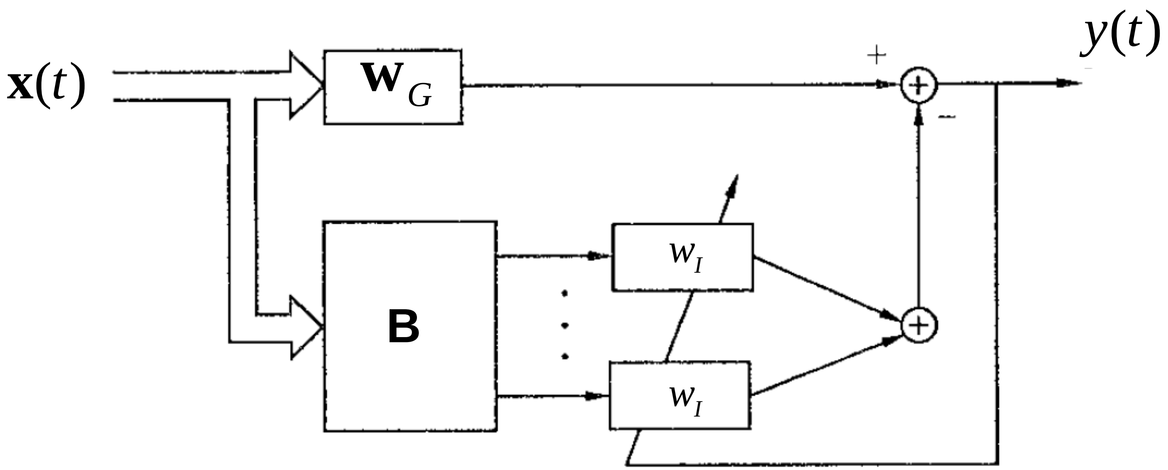


Adaptive Beamforming

- Another interpretation: Decompose $\mathbf{w}$ into

$$\mathbf{w} = \mathbf{w}_G - \mathbf{B}\mathbf{w}_I$$

$\mathbf{B}$ is the orthogonal compliment of $\mathbf{C}$: $\quad \mathbf{C}^H\mathbf{B} = \mathbf{0}$



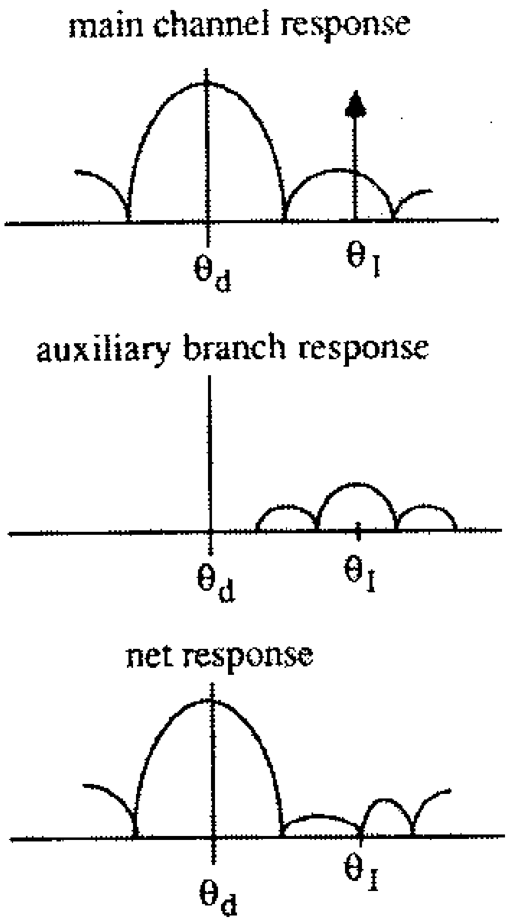
- Upper branch: Main channel; Lower branch: Auxiliary channel

Adaptive Beamforming

$$\mathbf{w}_G = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{g} \text{ so that } \mathbf{C}^H\mathbf{w}_G = \mathbf{g}$$

- I.e., $\mathbf{w}_G$ allows gain (null) in the desired directions – main channel
- Since $\mathbf{C}^H\mathbf{B} = \mathbf{0}$, the matrix **B** is a "blocking matrix" that blocks the signal (null) in the desired directions – auxiliary channel
- $\mathbf{w}_I$ is designed to produce a replica of interference leaking into the main channel to subtract it out

$$\mathbf{w}_I = (\mathbf{B}^H\mathbf{R}_{\mathbf{x}}\mathbf{B})^{-1}\mathbf{B}^H\mathbf{R}_{\mathbf{x}}\mathbf{w}_G$$



Adaptive Beamforming

- Advantages of adaptive beamforming
  - Able to reduce effect of interferences from unknown angles
  - Can steer look direction and multiple null directions
  - Get a signal copy easily

- Shortcomings of adaptive beamforming
  - Spatial resolution is still low
  - Need enough weights (antenna channels) to obtain enough degrees of freedom: At least # of constraints + 1
  - Sidelobe levels may not be controlled perfectly

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Multipath Propagation Issue

- Urban areas have a lot of multipaths
- Beamforming DOA algorithms are not affected by this much
- MUSIC fails in multipath!
- This is because if there is multipath, two or more DOAs will be from the same source – i.e., some sources in MUSIC signal model are correlated
- Now with I DOAs there are less than I sources

Multipath Propagation Issue

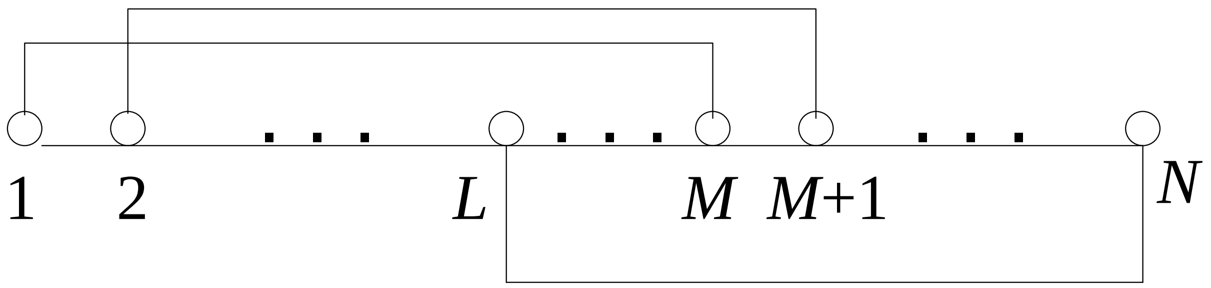
- Correlated sources disable MUSIC!
- E.g.: $\mathbf{s}(t) = [s_1(t), as_1(t)]^T$

$$\mathbf{R_s} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\} = \begin{bmatrix} \sigma_1^2 & a\sigma_1^2 \\ a\sigma_1^2 & a^2\sigma_1^2 \end{bmatrix} \Rightarrow \text{Rank } 1$$

- This is called rank deficiency
  - The rank of $\mathbf{AR_sA}^H$ will be less than I
  - The signal subspace has dimension less than I
  - There are less than I peaks in the MUSIC spectrum
  - Which of the I DOAs will give less than I peaks?
- In fact all peaks will be wrong.

Multipath Propagation Issue

- Remedy: Spatial Smoothing, still find I DOAs
  - L overlapping subarrays
  - M sensors in each subarray, $M > I$
  - N total sensors
  - $N = M + L - 1$



- Let $\mathbf{x}_i(t)$ be the received signal vector of the ith subarray

Multipath Propagation Issue

- One signal case:

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_N(t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-j\omega\frac{d\sin\theta}{c}} \\ e^{-j\omega\frac{2d\sin\theta}{c}} \\ \vdots \\ e^{-j\omega\frac{(N-1)d\sin\theta}{c}} \end{bmatrix} s(t) = \mathbf{a}(\theta)s(t)$$

➡ $\mathbf{x}_1(t) = \mathbf{a}_M(\theta)s(t),\ \mathbf{x}_2(t) = e^{-j\omega\frac{d\sin\theta}{c}}\mathbf{a}_M(\theta)s(t),$ etc.

- In general $\mathbf{x}_i(t) = e^{-j\omega\frac{(i-1)d\sin\theta}{c}}\mathbf{a}_M(\theta)s(t)$

Multipath Propagation Issue

- With I DOAs:

$$\mathbf{x}_i(t) = e^{-j\omega\frac{(i-1)d\sin\theta_1}{c}}\mathbf{a}_M(\theta_1)s_1(t) + \cdots + e^{-j\omega\frac{(i-1)d\sin\theta_I}{c}}\mathbf{a}_M(\theta_I)s_I(t)$$

$$= \underbrace{\begin{bmatrix}\mathbf{a}_M(\theta_1) & \cdots & \mathbf{a}_M(\theta_I)\end{bmatrix}}_{\mathbf{A}_M} \underbrace{\begin{bmatrix} e^{-j\omega\frac{(i-1)d\sin\theta_1}{c}} & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & e^{-j\omega\frac{(i-1)d\sin\theta_I}{c}} \end{bmatrix}}_{\mathbf{D}_i} \mathbf{s}(t)$$

➡ $\mathbf{x}_i(t) = \mathbf{A}_M\mathbf{D}_i\mathbf{s}(t)$ (Noise has been ignored so far)

Multipath Propagation Issue

- Compute correlation matrix of each subarray

$$\mathbf{R}_{\mathbf{x}i} = E\{\mathbf{x}_i(t)\mathbf{x}_i^H(t)\} = \mathbf{A}_M \mathbf{D}_i \mathbf{R}_\mathbf{s} \mathbf{D}_i^H \mathbf{A}_M^H + \sigma_0^2 \mathbf{I}$$

- Now average the correlation matrices of all subarrays

$$\mathbf{R}_{\mathbf{x}L} = \frac{1}{L}\sum_{i=1}^{L} E\{\mathbf{x}_i(t)\mathbf{x}_i^H(t)\} = \mathbf{A}_M \left[\frac{1}{L}\sum_{i=1}^{L} \mathbf{D}_i \mathbf{R}_\mathbf{s} \mathbf{D}_i^H\right] \mathbf{A}_M^H + \sigma_0^2 \mathbf{I}$$

- Note the dimension of $\mathbf{A}_M$ is M by I
- The dimension of the matrix in the brackets is I by I

Multipath Propagation Issue

$$\mathbf{R}_{\mathbf{x}L} = \frac{1}{L}\sum_{i=1}^{L} E\{\mathbf{x}_i(t)\mathbf{x}_i^H(t)\} = \mathbf{A}_M\left[\frac{1}{L}\sum_{i=1}^{L}\mathbf{D}_i\mathbf{R}_\mathbf{s}\mathbf{D}_i^H\right]\mathbf{A}_M^H + \sigma_0^2\mathbf{I}$$

- The matrix in the brackets has full rank *I* if *L* is large enough, i.e., $L \geq I$
- Provided *M* > *I*, there is a noise subspace in $\mathbf{R}_{\mathbf{x}L}$
- Now can apply MUSIC to $\mathbf{R}_{\mathbf{x}L}$
- Will detect *I* DOAs with this spatial smoothing
- Price paid is that more sensors are required: *M* > *I*, $L \geq I$

$M + L = N$ ⇨ *L* more sensors to “de-correlate”

Outline

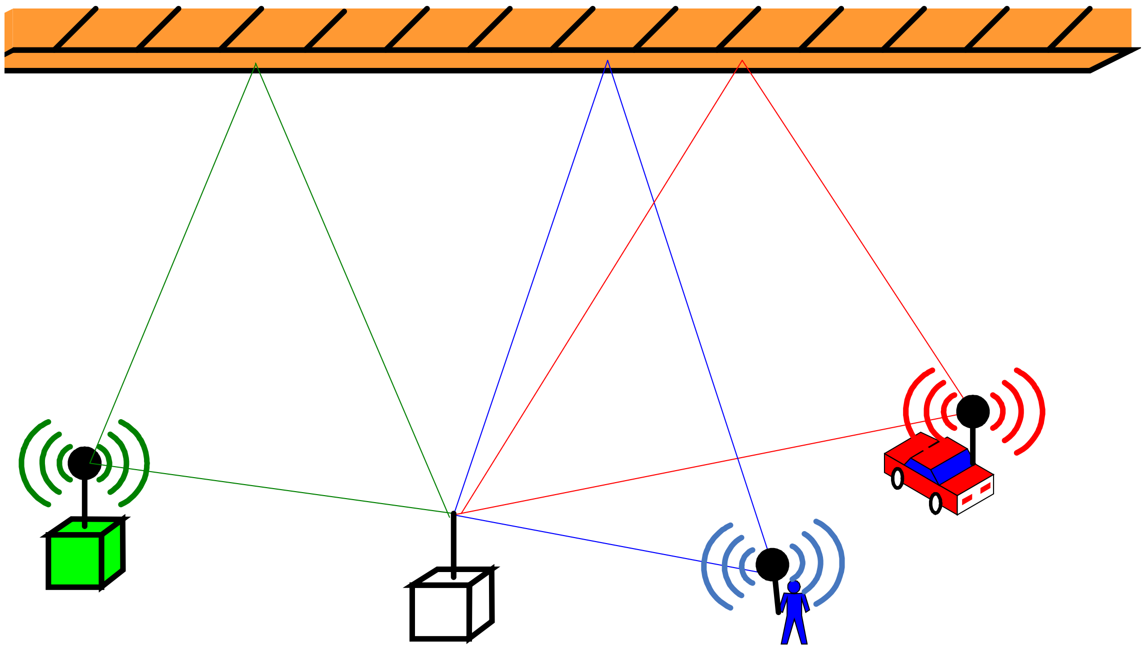
- Basics of DOA Estimation
  - Beamforming Type
  - High Resolution Type
- Adaptive Beamforming DOA Estimation in Strong Interference
- High Resolution DOA Estimation in Multipath
  - Smoothing Methods
- Source Association and Locating the Sources

Source Association Under Multipath

- Several paths belong to one source, multiple sources
- Can detect all DOAs of multiple paths and multiple sources
- But how many sources are there?
- Need to know which DOAs can be associated to which sources – source association
- Then locate the sources in several ways

Source Association Under Multipath

- Once sources are associated with DOAs
  - Locate the sources by **using** multipath if we know the reflection geometry – back tracing
- Multipath works to our advantage in this case



Source Association Under Multipath

- Alg. works for any DOA method, easier with MUSIC

1. Compute the received signal correlation matrix based on all sensors, as in MUSIC but un-smoothed
2. Compute the noise subspace matrix $\mathbf{U_n}$ whose rank is the number of sources *J*, not the number of paths *I*
3. Based on the estimated DOAs, compute the overall steering matrix **A**
4. Find minimum eigenvalues of $\mathbf{A}^H \mathbf{U_n} \mathbf{U_n}^H \mathbf{A}$ and their corresponding eigenvectors $\mathbf{q}_i$, the number of these eigenvalues is the number of sources *J*
5. Construct *I* by *J* matrix $\mathbf{Q} = [\mathbf{q}_1 \quad \cdots \quad \mathbf{q}_J]$

Source Association Under Multipath

6. Find J groups of rows of $\mathbf{Q} = [\mathbf{q}_1 \quad \cdots \quad \mathbf{q}_J]$ that are independent among the groups but dependent within each group, these correspond to groups of paths associated with each source

- Dependency test can be done by, e.g., dividing the corresponding elements in two rows and compute the variance of the ratios

Source Association Under Multipath

- E.g.: 15 antennas in a ULA, two sources, 5 paths
  - DOA from Source 1: -30^{o} and -60^{o}
  - DOA from Source 2: -5^{o}, 25^{o}, and 55^{o}
  - We only know 5 DOAs rank ordered as:

$$-60^{\text{o}},\ -30^{\text{o}},\ -5^{\text{o}},\ 25^{\text{o}},\ 55^{\text{o}}$$

- Put them into **A** in this order
- Computed eigenvalues of $\mathbf{A}^H \mathbf{U_n} \mathbf{U_n}^H \mathbf{A}$ which are {20.0559, 17.9585, 14.8226, 0.0430, 0.0088}
- Obviously $I = 5$ but $J = 2$

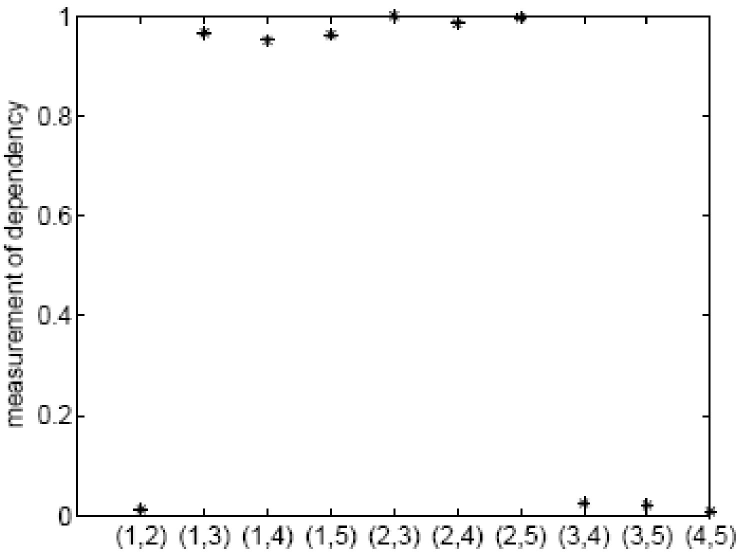
Source Association Under Multipath

$$\mathbf{Q} = [\mathbf{q}_1 \quad \mathbf{q}_2] = \begin{bmatrix} 0.4152 + j0.4824 & 0.2656 + j0.1121 \\ 0.4062 + j0.4989 & 0.2913 + j0.1122 \\ -0.2290 + j0.0746 & 0.3954 - j0.3430 \\ -0.2429 + j0.0498 & 0.3883 - j0.3502 \\ -0.2210 + j0.1123 & 0.2770 - j0.4418 \end{bmatrix}$$

- Dependency test on the rows of **Q**

➡ 1 & 2 belong to one

➡ 3, 4, 5 belong to the other

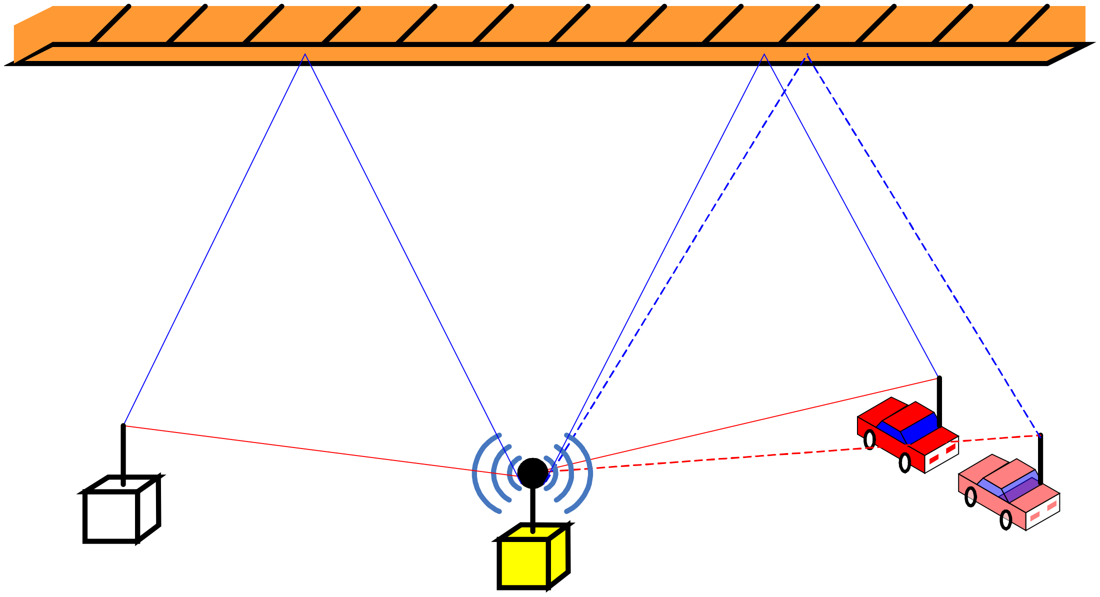


Source Association Under Multipath

- When reflection geometry is unknown
  - Identify the LOS paths from the reflected paths if either the source or the receiver is moving
- Once LOS paths are identified, the sources can be more easily located by
  - Ray back tracing
  - Several different angles of LOS DOA to triangulate

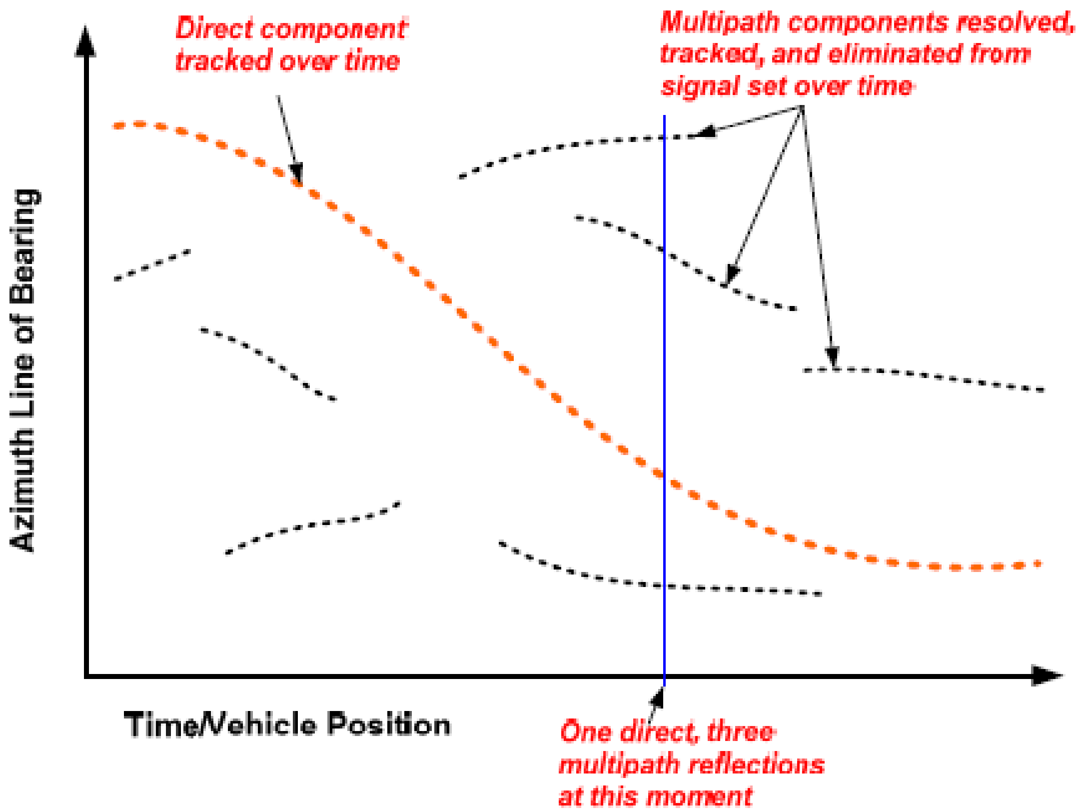
Source Association Under Multipath

- Stationary receiver: All paths have fixed DOAs;
- Moving receiver: Paths have varying DOAs, DOA variations differ between LOS path and reflected paths.



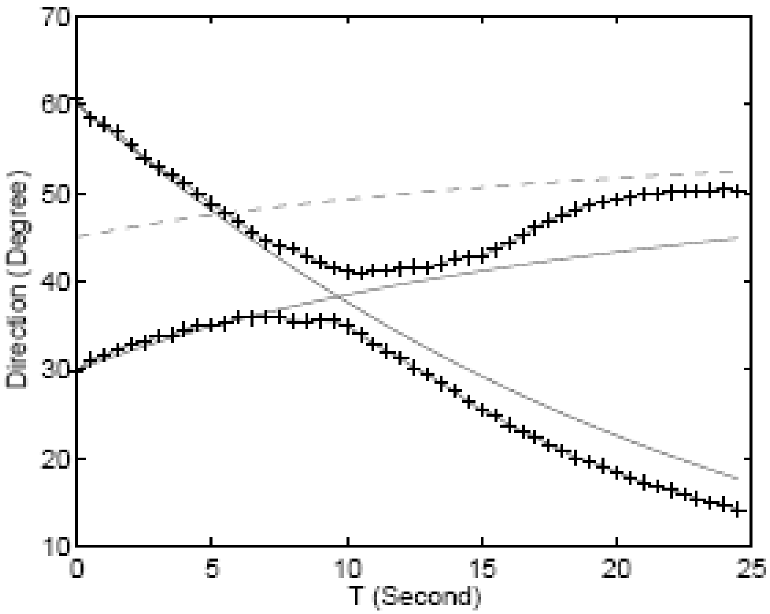
Source Association Under Multipath

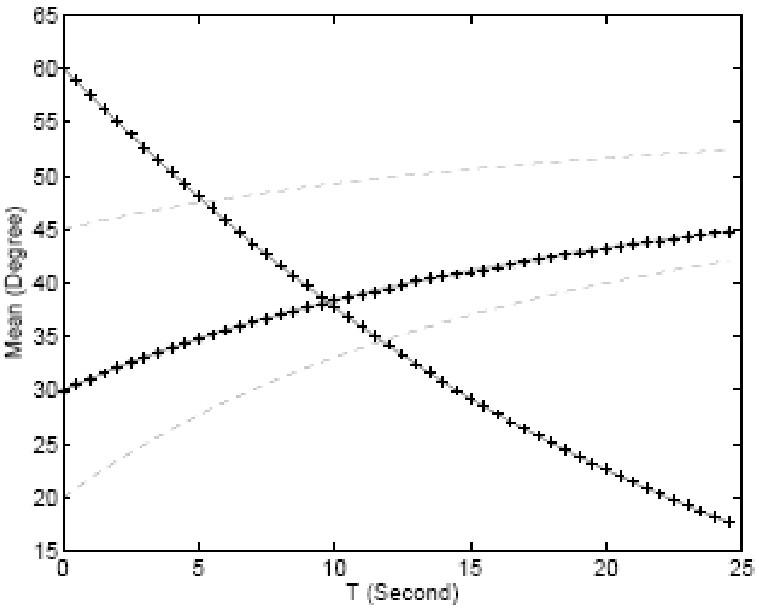
- Most likely, reflected paths appear and disappear intermittently as one travels, but not the LOS paths
- All DOAs need to be tracked
- Reflected paths may be eliminated over time by choosing the longest continuous path as the LOS path



Source Association Under Multipath

- Caution: If not tracked properly, two DOAs may be erroneously identified after they cross
- Special measures are needed to prevent this





Conclusions

- Beamforming (adaptive)
  - Can reduce effect of interferences from unknown DOAs
  - Can steer look direction and multiple null directions
  - Spatial resolution is low
  - Resilient to multipath propagation
- MUSIC (with smoothing)
  - Can find multiple DOAs with high resolution
  - Very sensitive to sensor position, gain, and phase errors, need careful calibration to make it work well
  - Spatial smoothing is difficult to achieve on other than ULA
- Source association can be applied to both methods
- Source location done by ray back trace/triangulation

About GIRD Systems, Inc.

- Founded in 2000 - based in Cincinnati
- Specializes in communications and signal processing, especially developing novel algorithms to solve challenging problems
- As of Jan. 2010, won more than 15 Phase I awards and 7 Phase II awards from Navy, Air Force, Army
- Partnerships with many large contractors including Northrop Grumman, L-3 Communications, etc.

About GIRD Systems, Inc.

- Key Technology Areas
  - Interference Mitigation (no reference, in-band)
  - Direction Finding (wideband, high-resolution)
  - Location/Navigation (Assisted GPS, GPS denied, signals of opportunity)
  - Wireless Network Security (physical layer)
  - Power Amplifier Linearization
  - Novel Communications systems/modeling
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